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INTERNATIONAL JOURNAL OF ADVANCED PRODUCTION AND INDUSTRIAL ENGINEERING

IJAPIE-2018-07-321, Vol 3 (3), 17-28

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FUZZY-STOCHASTIC MULTI-OBJECTIVE SUPPLY CHAIN COORDINATION MODELS

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Abstract : In this paper, we propose composite stochastic measure and fuzzy dominance degree to de-randomize and de-fuzzify fuzzy stochastic variables. These measures are then applied to solve the fuzzy stochastic profit sharing in long-term contracts for supply chain coordination problem with fuzzy cost, stochastic demand, and stochastic budget. These measures help to cover all shape characteristics as well as reflect the relative relationship between fuzzy numbers. We develop a new multi-item multi-objective manufacturer-retailer supply chain model that composes variance and expected profit in the objectives instead of setting allowance risk tolerances in the constraints. These allowances are easy to be biased. The proposed approach gives better results in both terms of supply chain profit and budget utilization. In addition, we extend our model to the more general case of all fuzzy stochastic cost, demand, and budget. The obtained solutions are very promising.

Keywords: supply chain coordination, profit sharing, fuzzy random variables.

1. INTRODUCTION

In recent developments, supply chain management studies have shown that sharing information among supply chain members is not enough to coordinate efficiently supply chain. A good system that shares profit to all supply chain members could give overall benefit for the whole supply chain (Wang, 2002; Yaghin et al., 2012; Saha et.al., 2015). The reasons of this fact could come from many factors such as uncertainty and imprecise demand, cost, and budget. The uncertain time of demand realization causes high risk to the members of supply chain. The demand realization experiences at the retailers will force the upstream members such as manufacturer to adjust its capacity. The additional cost of this adjustment bears unfairly to the manufacturer. Therefore, if we do not have a good coordination scheme to transfer certain portion profit from the retailer to the manufacturer, the supply of products to meet the additional demand will not satisfy customers in both terms of quantity and time. Some approaches have used buyback contracts to handle the impact of demand fluctuation that helps supply chain members to determine an appropriate adjustment strategy (Hu et al., 2010; Zang et al., 2014; He et al., 2016). In the study of Hu et al. (2010), the whole supply chain profit can be achieved by manufacturers repurchase strategy to deal with fuzzy random demand and imperfect quality products. Zang et al. (2014) also represented uncertain and fuzzy demand by a two-level buyback contract for a newsvendor model with a single cycle. The expected profit is defuzzified by using a crisp possibilistic mean value. The optimal order quantities in decentralized and centralized systems are analyzed and the conditions for supply chain coordination are obtained. However, Saha et al. (2015) stated that buyback contract is not effective when demand is realized before the retailer places its orders and after the manufacturer creates its capacity. They concentrate on the differences between the manufacturer's capacity and the final demand. Thus, the long-term contract with a profit-sharing scheme could be an effective mechanism to increase the marginal benefit for the manufacturer after increasing their capacity. From this viewpoint, Saha et al. (2015) proposed a multi-item multi-objective manufacturer–retailer supply chain model for long-term contracts with a profit-sharing scheme in fuzzy stochastic environment.

In addition, manufacturing cost and price are also influenced factors to supply chain coordination. Xu and Zhai (2010) designed a mechanism for coordinating single-period supply chain with fuzzy demand by setting different wholesale prices for different order quantities. With this pricing policy, the manufacturer can induce the retailer to order the right quantity and fully realize the maximum supply chain profit. They also found that both manufacturer and retailer can benefit by setting sharing proportion at an appropriate interval.

Besides cost and demand, budget availability and risk aversion factor are also concerned in supply chain decisions. Some studies investigated risk aversion impacts on the supply chain under some aspects of fuzzy and/or random budget and other parameters. Wu et al. (2013) developed a fuzzy stochastic multi-objective programming model for supply chain outsourcing risk management in presence of both random uncertainty and fuzzy uncertainty. Their results indicated that a more risk-averse customer prefers to order less under uncertainty and risk. Afrouzy at el. (2016) designed a fuzzy stochastic multi-echelon, multi-objective supply chain model incorporating new product development. In this case, the supplier capacity parameters of the supply chain and demand fluctuations are subject to uncertainty and imprecise. In their developed supply chain coordination models, Saha et al. (2015) also included risk and stochastic budget in the constraints with given level of acceptable risk tolerances for both manufacturer and retailer. These tolerances are easy to be biased by the decision maker. The separated treatment of risk aversion constraints could not minimize the affected risk by demand variation for both manufacturer and retailer. In addition, Saha et al. (2015) utilized the credibility measure to de-fuzzify fuzzy manufacturing cost by single expected value. This de-fuzzifying method does not reflect the relative relationship between fuzzy numbers with all shape characteristics.

In this paper, we develop new fuzzy stochastic supply chain coordination models for long-term contracts with a profit-sharing scheme. In the proposed models, we incorporate the demand variation in the objectives to minimize highest risk aversion factors and avoid decision makers bias. We also define new composite stochastic measure and fuzzy dominance degree to de-randomize and de-fuzzify fuzzy and/or stochastic variables. By these measures, the relative relationship between fuzzy numbers will be covered for all shape characteristics. Finally, we extend our development for the complete fuzzy stochastic supply chain coordination model for long-term contracts with a profit-sharing scheme in which all demand, manufacturing cost, and budget are fuzzy stochastic variables. Using our developed measures, the models are converted to the corresponding deterministic multi-objective linear programming model and solved by fuzzy compromise programming method, global criteria method and weighted sum method for comparison.

The paper will be presented as follows. In the next section, we will summarize some important results and define new measures for defuzifying and derandomizing fuzzy and/or stochastic variables. In Section 3, new supply chain coordination models for long-term contracts with profit sharing scheme are formulated and transformed to the equivalent deterministic ones. Section 4 will present numerical examples to illustrate the efficiency of the new models. Conclusion and future research direction are discussed in the final section.

2. FOUNDATION

There are several definitions of fuzzy random variables (Kruse and Meyer, 1987; Luhandjula, 1996; Couso and Dubois, 2015, etc.). Similar to the works of Montes et al. (2015), we define the fuzzy random variables as follows:

Definition 1 (Kruse and Meyer,1987): *Let* $(\Omega, \mathcal{A}, P)$ be a probability space. A fuzzy random variable is a map $\tilde{X}: \Omega \to \mathcal{F}(\mathbb{R})$ such that the α-cuts $\tilde{X}_\alpha: \Omega \to \mathcal{P}(\mathbb{R})$ given by:

$$\tilde{X}_\alpha(w) = \{t \in \mathbb{R}: \tilde{X}(w)(t) \geq \alpha\} \quad (1)$$

Are random sets, meaning that:

$$\{w \in \Omega: \tilde{X}_\alpha(w) \cap A \neq \emptyset\} \in \mathcal{A} \; \forall A \in \beta_{\mathbb{R}} \quad (2)$$

Different ranking methods for fuzzy random variables have been developed in the literature. The main difference between these methods is the way to defuzzify and derandomize fuzzy stochastic variables. The first direction is to perform the conversions (de-fuzzify, de-randomize) in a sequential manner (Luhandjula, 1996; Luhandjula and Gupta, 1996; Luhandjula, 2004; Yao and Wu, 2000). The second way is to perform both actions at the same time. (Liu 2001a, 2001b; Liu and Liu, 2002; Liu and Liu 2003, Aiche and Dubois, 2010; Montes et al., 2015; Couso and Dubois, 2015). The main disadvantage of the sequential method is to create a large number of additional variables. In the simultaneous approaches, both defuzzifying and derandomizing processes have been performed at the same time. Some approaches compare between fuzzy random variables by the expected value (Liu 2001a, 2001b; Liu and Liu, 2002; Liu and Liu 2003). The others extend the theory of expected utility and stochastic dominance to the comparison of fuzzy random variables (Aiche and Dubois, 2010; Montes et al., 2015; Couso and Dubois, 2015). Although these methods could cover different views of fuzzy random variables and offer a systematic approach for the combinations of stochastic and interval or fuzzy interval ranking, but the computation process is quite complicated. In this paper, the proposed method still follows the sequential method but the new measures could reduce significantly computational efforts. The proposed measures could also capture all probabilistic and shape information of fuzzy random variables (expected and variance values) to avoid the highly sensitivity expert dependence. In addition, the proposed approach uses the relative relationship instead of absolute values between the converted points of fuzzy stochastic variables.

Now, to handle the stochastic nature of fuzzy stochastics variables, we start from the similar combination of expectation and variance in the mathematical economics of Markowitz's model as used in the work of fuzzy stochastic OWA of Zarghami et al. (2008).

Definition 2: given a fuzzy random variable $\tilde{\tilde{x}}$, the stochastic nature of a fuzzy stochastic variable will be represented by the composite stochastic measure $\tilde{C}$ as follows:

$$\tilde{C} = E(\tilde{\tilde{x}}) - w\sqrt{Var(\tilde{\tilde{x}})}; 1 \geq w \geq 0 \quad (3)$$

Where

$$E(\tilde{\tilde{x}}) = \int_{-\infty}^{+\infty} \tilde{\tilde{x}} f(\tilde{\tilde{x}}) d\tilde{\tilde{x}} = \sum_{i=1}^{n} \tilde{\tilde{x}}_i f(\tilde{\tilde{x}}_i) \quad (4)$$

$$Var(\tilde{\tilde{x}}) = \int_{-\infty}^{+\infty} \left(\tilde{\tilde{x}} - E(\tilde{\tilde{x}})\right)^2 f(\tilde{\tilde{x}}) d\tilde{\tilde{x}} = \sum_{i=1}^{n} \left(\tilde{\tilde{x}}_i - E(\tilde{\tilde{x}})\right)^2 f(\tilde{\tilde{x}}_i) \quad (5)$$

where w is a positive weight showing the importance of decreasing risk in comparison to maximizing the expected payoff. It is clear that the composite stochastic measure $\tilde{C}$ is a fuzzy number.

Next, to compare two fuzzy numbers, we develop a new concept of fuzzy dominance and its associated measure, fuzzy dominance degree. Let $\succ$ be the binary relation on fuzzy set X, representing "more dominated than". Let $\approx$ be the binary relation "indifferent to" on fuzzy set X. Let $\prec$ be the binary relation on fuzzy set X, representing "less dominated than". The following will define how a fuzzy number dominates than another one:

Definition 3 (fuzzy domination): Given two fuzzy numbers $\tilde{P}, \tilde{Q} \in X$, we have:

$$\tilde{P} \succ \tilde{Q} \text{ if and only if } P^{\alpha} > Q^{\alpha}; 0 \le \alpha \le 1$$
$$\tilde{P} \approx \tilde{Q} \text{ if and only if } P^{\alpha} = Q^{\alpha}; 0 \le \alpha \le 1$$
$$\tilde{P} \prec \tilde{Q} \text{ if and only if } P^{\alpha} < Q^{\alpha}; 0 \le \alpha \le 1$$

if $\tilde{P} \succ \tilde{Q}$ we say that $\tilde{P}$ dominates than $\tilde{Q}$. To measure how much $\tilde{P}$ dominates than $\tilde{Q}$, let $D(\tilde{P},\tilde{Q})$ be the dominance degree of $\tilde{P}$ over $\tilde{Q}$, we have:

Definition 4: Given two fuzzy number $\tilde{P}, \tilde{Q} \in X_w$

$$D(\tilde{P},\tilde{Q}) > 0 \text{ if and only if } \tilde{P} \succ \tilde{Q}$$
$$D(\tilde{P},\tilde{Q}) = 0 \text{ if and only if } \tilde{P} \approx \tilde{Q}$$
$$D(\tilde{P},\tilde{Q}) < 0 \text{ if and only if } \tilde{P} \prec \tilde{Q}$$

$D(\tilde{P},\tilde{Q})$ somehow indicates the strength of preference between fuzzy numbers. To formalize the detailed calculation of $D(\tilde{P},\tilde{Q})$, we start from the α-cut of two fuzzy numbers $\tilde{P}$ and $\tilde{Q}$ (see Fig. 1):

$$\tilde{P}_{\alpha} = \mu_{\tilde{P}}(x) \ge \alpha$$
$$\tilde{Q}_{\alpha} = \mu_{\tilde{Q}}(x) \ge \alpha \qquad (6)$$

We can see that if $\tilde{P}_{\alpha} \ge \tilde{Q}_{\alpha}$ then $\sup\{s : \mu_{\tilde{P}}(s) \ge \alpha\} - \sup\{s : \mu_{\tilde{Q}}(s) \ge \alpha\} \ge 0$. This amount indicates the preference of a fuzzy number over another one. Therefore, the total preference degree of $\tilde{P}$ over $\tilde{Q}$ is the area of $(\tilde{P} > \tilde{Q}) + (\tilde{Q} = \tilde{P})$. Mathematically, the fuzzy dominance degree is defined as follows.

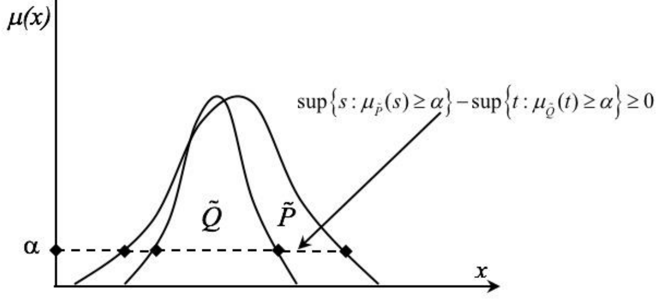

Fig.1 The preference level at α-level

Definition 5: (**fuzzy dominance degree**) for $\tilde{P}, \tilde{Q}$, we have

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_{\alpha} d\alpha = \int_0^1 \left\{(\tilde{P} > \tilde{Q})_{\alpha} + (\tilde{Q} = \tilde{P})_{\alpha}\right\} d\alpha =$$
$$\int_0^1 \left\{\left\{max\left\{0, sup\{s: \mu_{\tilde{P}}(s) \ge \alpha\} - sup\{t: \mu_{\tilde{Q}}(t) \ge \alpha\}\right\} + max\left\{0, min\left\{sup\{t: \mu_{\tilde{Q}}(t) \ge \alpha\}, sup\{s: \mu_{\tilde{P}}(s) \ge \alpha\}\right\} - max\left\{inf\{s: \mu_{\tilde{P}}(s) \ge \alpha\}, inf\{t: \mu_{\tilde{Q}}(t) \ge \alpha\}\right\}\right\}\right\}\right\} d\alpha$$

(7)

For simplicity in calculation, let consider $\tilde{T}$ be a family of triangular fuzzy numbers represented by a triplet (τ,a,b) with its membership function is defined by (Zimmerman, 1985) :

$$\mu_{\tilde{T}}(x) = \begin{cases} \max\left(0, 1 - \dfrac{\tau - x}{a}\right); \text{if } x \le \tau, a > 0 \\ 1, \text{ if } a = 0 \text{ and/or } b = 0 \\ \max\left(0, 1 - \dfrac{x - \tau}{b}\right); \text{if } x > \tau, b > 0 \\ 0; \text{otherwise} \end{cases} \qquad (8)$$

where the scalars a, b ≥ 0 (a, b $\in \mathfrak{R}$) are called the left and right spreads, respectively. This type of fuzzy numbers is quite popular and allows quantification of quite different types of information. A crisp number $\tau \in \mathfrak{R}$ can be illustrated as a triangular fuzzy number $\tilde{\tau} = (\tau, 0, 0)$.

Now, if two triangle fuzzy numbers $\tilde{P}, \tilde{Q}$ mentioned above are triangle fuzzy numbers represented by triplet values $\tilde{P}(u,a,b)$ and $\tilde{Q}(v,c,d)$, the α-cut of $\tilde{P}$ and $\tilde{Q}$ are

$$[P^L(\alpha), P^U(\alpha)] = [u - a(1-\alpha), u + b(1-\alpha)]$$
$$[Q^L(\alpha), Q^U(\alpha)] = [v - c(1-\alpha), v + d(1-\alpha)] \qquad (6)$$

From (7), the fuzzy dominance degree is determined by the different areas between $\tilde{P}$ and $\tilde{Q}$ such that $(\tilde{P} > \tilde{Q})$ and $(\tilde{P} = \tilde{Q})$. Similar to the results of Lin et al. (2017), we investigate the following cases of the relative positions between $\tilde{P}$ and $\tilde{Q}$.

Case 1 (see Fig. 2): $v < u, u - a \le v - c \le u + b \le v + d$

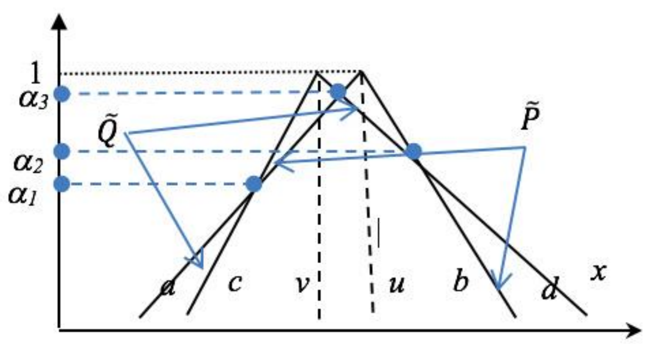

Fig.2 Case of $(v < u, u - a \le v - c \le u + b \le v + d)$

In this case, we have:

$$\alpha_1 = 1 + \frac{v-u}{a-c}; \alpha_2 = 1 + \frac{v-u}{d-b}; \alpha_3 = 1 + \frac{v-u}{d+a};$$

The dominance degree $D(\tilde{P},\tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha$$
$$= \int_{\alpha_2}^{1} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^U)d\alpha + \int_0^{\alpha_1} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^L)d\alpha$$
$$+ \int_{\alpha_1}^{\alpha_2} (\tilde{P}_\alpha^U - \tilde{P}_\alpha^L)d\alpha + \int_{\alpha_2}^{\alpha_3} (\tilde{Q}_\alpha^U - \tilde{P}_\alpha^L)d\alpha$$
$$= u - v + \frac{b-d}{2} + (u-v-a+c)\alpha_1$$
$$+ (a-c)\frac{\alpha_1^2}{2} + (u-v+a+d)\alpha_3 - (a$$
$$+ d)\frac{\alpha_3^2}{2}$$

Case 2 (see Fig. 3): $v < u, u-a \le v-c \le v+d \le u+b$.

In this case, we have:

$$\alpha_1 = 1 + \frac{v-u}{a-c}; \alpha_2 = 1 + \frac{v-u}{d+a};$$

The dominance degree $D(\tilde{P},\tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha$$
$$= \int_0^{1} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^U)d\alpha + \int_0^{\alpha_2} (\tilde{Q}_\alpha^U - \tilde{Q}_\alpha^L)d\alpha$$
$$+ \int_{\alpha_1}^{\alpha_2} (\tilde{Q}_\alpha^U - \tilde{P}_\alpha^L)d\alpha$$
$$= u - v + \frac{b-d}{2} + (u-v+c-a)\alpha_1$$
$$+ (a-c)\frac{\alpha_1^2}{2} + (v-u+a+|d)\alpha_2 - (a$$
$$+ d)\frac{\alpha_2^2}{2}$$

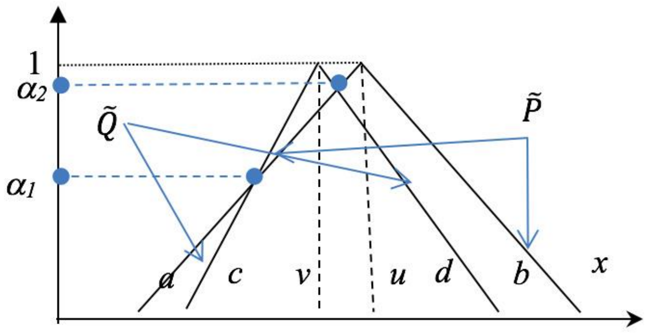

Fig.3 Case of $(v < u, u-a \le v-c \le v+d \le u+b)$

Case 3 (see Fig. 4): $v < u, v-c \le u-a \le v+d \le u+b$

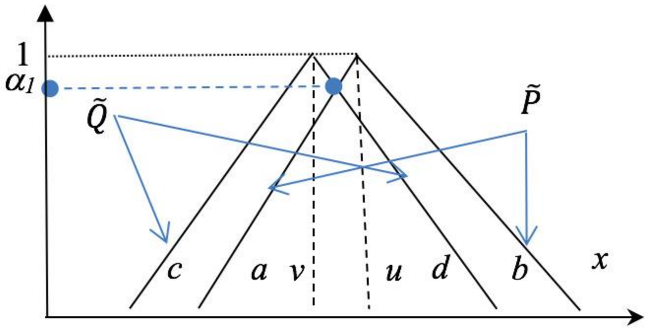

Fig.4 Case of $(v < u, v-c \le u-a \le v+d \le u+b)$

In this case, we have:
$\alpha_1 = 1 + \frac{v-u}{d+a}$;The dominance degree $D(\tilde{P},\tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha$$
$$= \int_0^{1} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^U)d\alpha + \int_0^{\alpha_1} (\tilde{Q}_\alpha^U - \tilde{P}_\alpha^L)d\alpha$$
$$= u - v + \frac{b-d}{2} + (v-u+a+d)\alpha_1$$
$$- (a+d)\frac{\alpha_1^2}{2}$$

Case 4 (see Fig. 5): $v < u, v-c \le u-a \le u+b \le v+d$

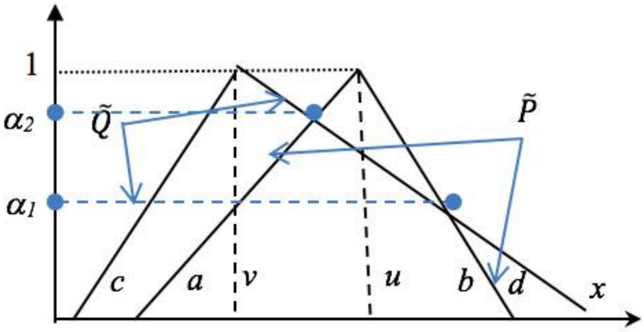

Fig.5 Case of $(v < u, v-c \le u-a \le u+b \le v+d)$

In this case, we have:

$$\alpha_1 = 1 + \frac{v-u}{d-b}; \alpha_2 = 1 + \frac{v-u}{d+a};$$

The dominance degree $D(\tilde{P},\tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha$$
$$= \int_{\alpha_1}^{1} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^U)d\alpha + \int_0^{\alpha_1} (\tilde{P}_\alpha^U - \tilde{P}_\alpha^L)d\alpha$$
$$+ \int_{\alpha_1}^{\alpha_2} (\tilde{Q}_\alpha^U - \tilde{P}_\alpha^L)d\alpha$$
$$= u - v + \frac{b-d}{2} + (v-u+a+d)\alpha_2$$
$$- (a+d)\frac{\alpha_2^2}{2}$$

Case 5 (see Fig. 6): $v < u, v - c \leq v + d \leq u - a \leq u + b$

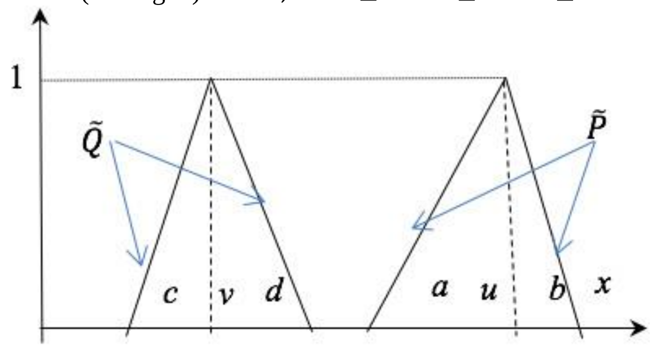

Fig.6 Case of $(v < u, v - c \leq v + d \leq u - a \leq u + b)$

In this case, the dominance degree $D(\tilde{P},\tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha = \int_0^1 (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^U) d\alpha$$

$$= u - v + \frac{b-d}{2}$$

Case 6 (see Fig. 7): $u \leq v, v - c \leq u - a \leq v + d \leq u + b$.

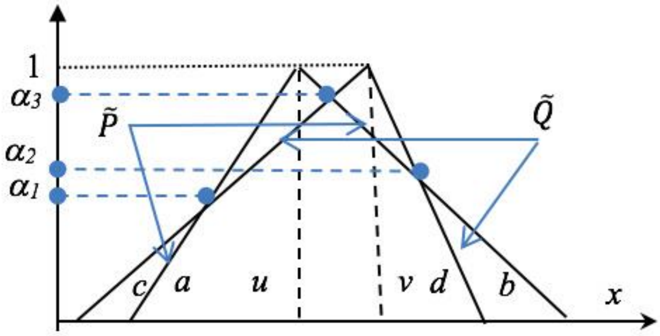

Fig.7 Case of $(u \leq v, v - c \leq u - a \leq v + d \leq u + b)$

In this case, we have:

$$\alpha_1 = 1 + \frac{v-u}{a-c}; \alpha_2 = 1 + \frac{v-u}{d-b}; \alpha_3 = 1 + \frac{u-v}{b+c};$$

The dominance degree $D(\tilde{P},\tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha$$

$$= \int_0^{\alpha_2} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^U) d\alpha + \int_0^{\alpha_1} (\tilde{Q}_\alpha^U - \tilde{P}_\alpha^L) d\alpha$$

$$+ \int_{\alpha_1}^{\alpha_2} (\tilde{Q}_\alpha^U - \tilde{Q}_\alpha^L) d\alpha + \int_{\alpha_3}^{1} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^L) d\alpha$$

$$= (v - u + a - c)\alpha_1 + (c - a)\frac{\alpha_1^2}{2}$$

$$+ (u - v + b + c)\alpha_3 - (b + c)\frac{\alpha_3^2}{2}$$

Case 7 (see Fig. 8): $u \leq v, v - c \leq u - a \leq u + b \leq v + d$.

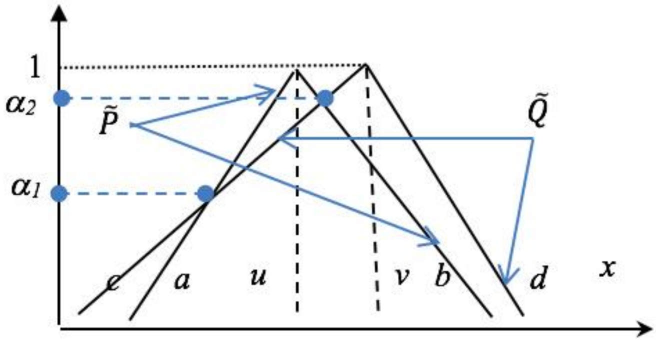

Fig. 8. Case of $(u \leq v, v - c \leq u - a \leq u + b \leq v + d)$

In this case, we have:

$$\alpha_1 = 1 + \frac{v-u}{a-c}; \alpha_2 = 1 + \frac{u-v}{b+c}$$

The dominance degree $D(\tilde{P},\tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha$$

$$= \int_0^{\alpha_1} (\tilde{P}_\alpha^U - \tilde{P}_\alpha^L) d\alpha + \int_{\alpha_1}^{\alpha_2} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^L) d\alpha$$

$$= (v - u + a)\alpha_1 + (c - a)\frac{\alpha_1^2}{2}$$

$$+ (u - v + c + b)\alpha_2 - (b + c)\frac{\alpha_2^2}{2}$$

Case 8 (see Fig. 9): $u \leq v, u - a \leq v - c \leq u + b \leq v + d$.

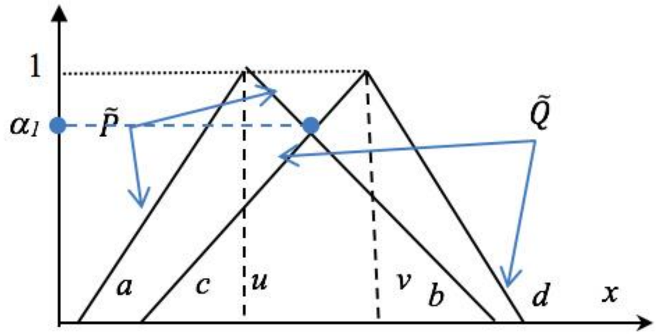

Fig. 9. Case of $(u \leq v, u - a \leq v - c \leq u + b \leq v + d)$

In this case, we have:

$$\alpha_1 = 1 + \frac{v-u}{d+a};$$

The dominance degree $D(\tilde{P},\tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha = \int_0^{\alpha_1} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^L) d\alpha$$

$$= (u - v + b + c)\alpha_1 - (b + c)\frac{\alpha_1^2}{2}$$

Case 9 (see Fig. 10): $u \leq v, u - a \leq v - c \leq v + d \leq u + b$.

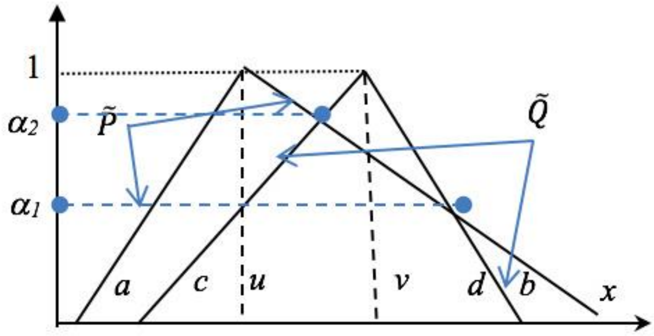

Fig. 10. Case of $(u \le v, u - a \le v - c \le v + d \le u + b)$

In this case, we have:

$$\alpha_1 = 1 + \frac{v-u}{d-b}; \alpha_2 = 1 + \frac{v-u}{d+a};$$

The dominance degree $D(\tilde{P}, \tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha = \int_0^{\alpha_2} (\tilde{P}_\alpha^U - \tilde{Q}_\alpha^L) d\alpha$$

$$= (u - v + b + c)\alpha_2 - \frac{b+c}{2}\alpha_2^2$$

Case 10 (see Fig. 11): $u \le v, u - a \le u + b \le v - c \le v + d$.

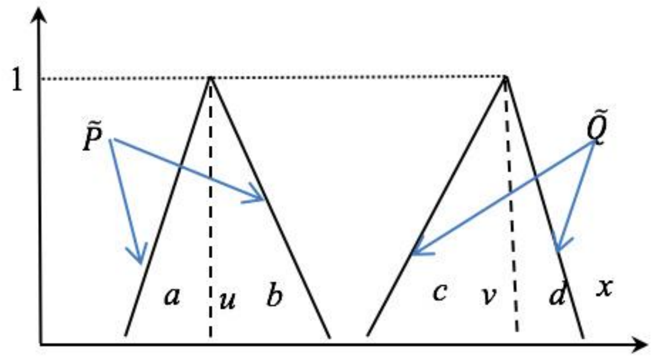

Fig. 11. Case of $(u \le v, u - a \le u + b \le v - c \le v + d)$

In this case, the dominance degree $D(\tilde{P}, \tilde{Q})$ of $\tilde{P}$ over $\tilde{Q}$ will be:

$$D(\tilde{P},\tilde{Q}) = \int_0^1 D(\tilde{P},\tilde{Q})_\alpha d\alpha = 0$$

With the definitions of composite stochastic measure and dominance degree, we can convert the fuzzy stochastic variables into the corresponding deterministic ones that cover all shape characteristics as well as reflect the relative relationships between fuzzy numbers. We are using these results to improve the supply chain coordination models for long-term contracts with profit sharing scheme including fuzzy manufacturing cost, stochastic demand, and stochastic budget in the following section.

3. FUZZY STOCHASTIC SUPPLY CHAIN COORDINATION WITH PROFIT SHARING

3.1. Profit Sharing Supply Chain Coordination with stochastic demand and fuzzy cost

In this section, we consider again the supply chain coordination with profit sharing problem stated in the work of Saha et al. (2015) in which the supply chain system includes two components: a manufacturer and a retailer. The retailer sells multiple types of items with stochastic demand.

We model the problem in different way by using the original fuzzy and/or stochastic variables instead of expected value and constrained the variance

$$Max\ Z_M = \sum_{i=1}^m n\left[(s_i - \tilde{c}_i)y_i + (1 - \alpha_i)\left[(r_i - s_i)y_i - (r_i - v_i)(y_i - \hat{d}_i)\right]\right] \quad (9)$$

$$Max\ Z_R = \sum_{i=1}^m n\left[\alpha_i\left[(r_i - s_i)y_i - (r_i - v_i)(y_i - \hat{d}_i)\right]\right] \quad (10)$$

Subject to

$$\sum_{i=1}^m s_i y_i \le \hat{B} \quad (11)$$

$$y_i \ge 0; i = 1,2,\dots,m \quad (12)$$

Where:

n: number of planning periods
r_i: the retail price per unit item, $i = 1, 2, \dots, m$
s_i: the selling price per unit item
v_i: The salvage value per unit item.
$\tilde{c}_i$: The fuzzy manufacturing cost per unit item.
α_i: The profit sharing fraction ($0 < a_i < 1$).
y_i: The ordering quantity of the item for each period.
$\hat{d}_i$: The random market demand
Z_M: The total expected profit of the manufacturer in n periods.
Z_R: The total expected profit of the retailer in n periods.
$\hat{B}$: The probabilistic budget for the retailer, which is a normal random variable.

To handle fuzzy and/or stochastic variables, we will isolate these variables and use the previous results to convert the model to the equivalent deterministic one. Model (9) - (12) will be:

$$Max\ Z_M = \sum_{i=1}^m n\left[-(\tilde{c}_i)y_i + (1 - \alpha_i)(r_i - v_i)\hat{d}_i\right] + \sum_{i=1}^m n\left[s_i y_i + (1 - \alpha_i)(v_i - s_i)y_i\right] \quad (13)$$

$$Max\ Z_R = \sum_{i=1}^m n\left[\alpha_i(r_i - v_i)\hat{d}_i\right] + \sum_{i=1}^m n\alpha_i\left[(v_i - s_i)y_i\right] \quad (14)$$

Subject to

$$\sum_{i=1}^m s_i y_i \le \hat{B} \quad (15)$$

$$y_i \ge 0; i = 1,2,\dots,m \quad (16)$$

First, we need to de-randomize our model by defining composite stochastic measures of demand $\hat{d}_i$ and budget $\hat{B}$ as follows

$$\delta_i = E(\hat{d}_i) - w_d\sqrt{Var(\hat{d}_i)};\ 1 \geq w_d \geq 0 \quad (17)$$

$$\beta = E(\hat{B}) - w_B\sqrt{Var(\hat{B})};\ 1 \geq w_B \geq 0 \quad (18)$$

The de-randomized model will be:

$$Max\ Z_M = \lambda_M \quad (19)$$

$$Max\ Z_R = \lambda_R \quad (20)$$

Subject to

$$\sum_{i=1}^m n[-(\tilde{c}_i)y_i + (1-\alpha_i)(r_i - v_i)\delta_i] + \sum_{i=1}^m n[s_i y_i + (1-\alpha_i)(v_i - s_i)y_i] \geq \lambda_M \quad (21)$$

$$\sum_{i=1}^m n[\alpha_i(r_i - v_i)\delta_i] + \sum_{i=1}^m n\alpha_i[(v_i - s_i)y_i] \geq \lambda_R \quad (22)$$

$$\sum_{i=1}^m s_i y_i \leq \beta \quad (23)$$

$$y_i \geq 0;\ i = 1,2,\dots,m \quad (24)$$

we are continuing to separate fuzzy numbers with scrip-values as follows

$$Max\ Z_M = \lambda_M \quad (25)$$

$$Max\ Z_R = \lambda_R \quad (26)$$

Subject to

$$\sum_{i=1}^m n[(1-\alpha_i)(r_i - v_i)\delta_i] + \sum_{i=1}^m n[s_i y_i + (1-\alpha_i)(v_i - s_i)y_i] - \lambda_M \geq \sum_{i=1}^m n(\tilde{c}_i)y_i \quad (27)$$

$$\sum_{i=1}^m n[\alpha_i(r_i - v_i)\delta_i] + \sum_{i=1}^m n\alpha_i[(v_i - s_i)y_i] \geq \lambda_R \quad (28)$$

$$\sum_{i=1}^m s_i y_i \leq \beta \quad (29)$$

$$y_i \geq 0;\ i = 1,2,\dots,m \quad (30)$$

Utilizing the concept of dominance degree in (7), we have:

$$Max\ Z_M = \lambda_M \quad (31)$$

$$Max\ Z_R = \lambda_R \quad (32)$$

Subject to

$$D(\tilde{\theta}_M, \sum_{i=1}^m n(\tilde{c}_i)y_i) \geq 0 \quad (33)$$

$$\sum_{i=1}^m n[\alpha_i(r_i - v_i)\delta_i] + \sum_{i=1}^m n\alpha_i[(v_i - s_i)y_i] \geq \lambda_R \quad (34)$$

$$\sum_{i=1}^m s_i y_i \leq \beta \quad (35)$$

$$y_i \geq 0;\ i = 1,2,\dots,m \quad (36)$$

Where: crisp values θ is equivalent to fuzzy number (θ, 0,0):

$$\tilde{\theta}_M = (\sum_{i=1}^m n[s_i y_i + (1-\alpha_i)(v_i - s_i)y_i + (1-\alpha_i)(r_i - v_i)\delta_i] - \lambda_M, 0,0) \quad (37)$$

Solving the equivalent deterministic model (31) – (37), we will obtain the optimal solution.

3.2. Profit Sharing Supply Chain Coordination with all fuzzy stochastic demand, cost, and budget

From the above results, we can also extend our model to the most general case of all fuzzy stochastic cost, demand, and budget. In such case, the supply chain coordination model for long-term contract with a profit-sharing scheme will be:

$$Max\ Z_M = \sum_{i=1}^m n\left[(s_i - \tilde{\hat{c}}_i)y_i + (1-\alpha_i)\left[(r_i - s_i)y_i - (r_i - v_i)(y_i - \tilde{\hat{d}}_i)\right]\right] \quad (38)$$

$$Max\ Z_R = \sum_{i=1}^m n\left[\alpha_i\left[(r_i - s_i)y_i - (r_i - v_i)(y_i - \tilde{\hat{d}}_i)\right]\right] \quad (39)$$

Subject to

$$\sum_{i=1}^m s_i y_i \leq \tilde{\hat{B}} \quad (40)$$

$$y_i \geq 0;\ i = 1,2,\dots,m \quad (41)$$

We also define the composite stochastic measures as follows:

$$\tilde{\delta}_i = E\left(\tilde{\hat{d}}_i\right) - w_d\sqrt{Var(\tilde{\hat{d}}_i)};\ w_d \geq 0 \quad (42)$$

$$\tilde{\beta} = E(\tilde{\hat{B}}) - w_B\sqrt{Var(\tilde{\hat{B}})};\ w_B \geq 0 \quad (43)$$

$$\tilde{\gamma}_i = E(\tilde{\hat{c}}_i) - w_c\sqrt{Var(\tilde{\hat{c}}_i)};\ w_c \geq 0 \quad (44)$$

Now, the model (38) – (41) will be derandomized as follows:

$$Max\ Z_M = \lambda_M \quad (45)$$

$$Max\ Z_R = \lambda_R \quad (46)$$

Subject to

$$\sum_{i=1}^m n[s_i y_i + (1-\alpha_i)[(r_i - s_i)y_i - (r_i - v_i)y_i]] - \lambda_M \geq \sum_{i=1}^m n[\tilde{\gamma}_i y_i + (1-\alpha_i)(v_i - r_i)\tilde{\delta}_i] \quad (47)$$

$$\sum_{i=1}^m n\alpha_i[(r_i - s_i)y_i - (r_i - v_i)y_i] - \lambda_R \geq \sum_{i=1}^m n\alpha_i(v_i - r_i)\tilde{\delta}_i \quad (48)$$

$$\sum_{i=1}^m s_i y_i \leq \tilde{\beta} \quad (49)$$

$$y_i \geq 0;\ i = 1,2,\dots,m \quad (50)$$

Finally, the defuzzifying model of (45) – (50) will be:

$$Max\ Z_M = \lambda_M \quad (51)$$

$$Max\ Z_R = \lambda_R \quad (52)$$

Subject to

$$D(\tilde{\theta}_M, \sum_{i=1}^m[\tilde{\gamma}_i y_i + (1-\alpha_i)(v_i - r_i)\tilde{\delta}_i]) \geq 0 \quad (53)$$

$$D(\tilde{\theta}_R, \sum_{i=1}^m \alpha_i(v_i - r_i)\tilde{\delta}_i) \geq 0 \quad (54)$$

$$D(\tilde{\beta}, \tilde{\theta}) \geq 0 \quad (55)$$

$$y_i \geq 0;\ i = 1,2,\dots,m \quad (56)$$

Where:

$$\tilde{\theta}_M = (\sum_{i=1}^m[s_i y_i + (1-\alpha_i)(v_i - s_i)y_i] - \lambda_M/n, 0,0) \quad (57)$$

$$\tilde{\theta}_R = (\sum_{i=1}^m \alpha_i(v_i - s_i)y_i - \lambda_R/n, 0,0) \quad (58)$$

$$\tilde{\theta} = (\sum_{i=1}^m s_i y_i, 0,0) \quad (59)$$

This model is the normal deterministic linear programming model. It is easily solved by standard tools such as LINGO or CPLEX.

4. NUMERICAL EXPERIMENT

First of all, we will compare our approach with the work of Saha et al. (2015). Then, we extend our investigation for the most general case of profit-sharing supply chain coordination model with all fuzzy stochastic demand, cost, and budget.

The results of Saha et al. (2015) is best obtained by using Fuzzy Compromise Programming Method (FCPM). Therefore, we will perform the comparison between our model and Saha's model II using the same FCPM method. For ease of reading, we summarize the FCPM method as follows:

Let Z_M^{min}, Z_M^{max}, Z_R^{min}, Z_R^{max} be the aspired level and the highest acceptable level of achievement for the single objective solution of the multi-objective linear programming(MOLP) model (while disregarding the other objectives). The FCPM model is based on maximizing the least satisfaction level among all objectives as follows:

$$Max\ \lambda \quad (60)$$

$$\frac{Z_M - Z_M^{min}}{Z_M^{max} - Z_M^{min}} \geq \lambda \quad (61)$$

$$\frac{Z_R - Z_R^{min}}{Z_R^{max} - Z_R^{min}} \geq \lambda \quad (62)$$

$$1 \geq \lambda \geq 0 \quad (63)$$

$$x \in X \quad (64)$$

Constraint (64) is a set of given constraints of Saha's model or our model.

Now, we are revisiting the Saha's example to investigate numerically the models. In that example, three different types of items (m = 3) are considered. The other parameters of the problem are:

n = 50; ρ= 0.4; α_1 = 0.55; r_1 = \$90; v_1 = \$6; s_1 = \$40;
c_{11} = \$7; c_{12} = \$10; c_{13} = \$13; →u_1 = 10; a_1 =3; b_1=3
α_2 = 0.53; r_2 = \$100; v_2 = \$12; s_2 = \$45;
c_{21} = \$13; c_{22} = \$15; c_{23} = \$17; →u_2 = 15; a_2 =2; b_2=2
α_3 = 0.51; r_3 = \$110; v_3 = \$14; s_3 = \$50;
c_{31} = \$15; c_{32} = \$16; c_{33} = \$17; →u_3 = 16; a_3 =1; b_2=1

Without loss of generality, we assume that the weighted value of the credibility measure in Saha and our composite stochastic measures are the same: $\rho = w_d = w_c = w_B = 0.4$. The probabilistic budget is normally distributed with $E(\hat{B}) = \$15{,}000, Var(\hat{B}) = \$700, and\ \Phi(R) = -1.2$. We also consider three popular types of probability distribution: uniform, exponential, and normal distributions with the same input parameters of Saha (2015) presented in the following table:

Table 1. Distribution and input parameters

Distribution	$E(x_i)$	$Var(x_i)$	Input Parameters
Uniform Distribution $f(x_i) = \frac{1}{b_i - a_i}, b_i \geq x_i \geq a_i$	$\left[\frac{a_i + b_i}{2}\right]$	$\left[\frac{(b_i - a_i)^2}{12}\right]$	a_1=0, b_1=120, a_2=0, b_2= 160, a_3 =0, b_3= 200
Exponential Distribution $f(x_i) = \beta_i e^{-\beta_i x_i}, x_i \geq 0$	$\frac{1}{\beta_i}$	$\left(\frac{1}{\beta_i^2}\right)$	β_1=0.015, β_2 = 0.011, β_3=0.009
Normal Distribution $f(x_i) = \frac{1}{\sigma_i\sqrt{2\pi}} e^{-\frac{(x_i-\mu_i)^2}{2\sigma_i^2}}$	μ_i	σ_i^2	μ_1= 60,σ_1=24, μ_2= 80, σ_2=32, μ_3= 100,σ_3=40

We are solving our model with the same Saha's data presented above. Our solution results and Saha's results by using FCPM are summarized in Table 2

Table 2. Solution of Saha's Model II and our Model

Solution Method	Max/Min Values and Optimal Variables	Type of distribution for stochastic demand		
		Uniform Distribution	Exponential Distribution	Normal Distribution
Saha's Model II	Z_M^{max}	688,548.5	612,166.8	688,213.3
	Z_M^{min}	669,017.7	556,240.1	668,929.9
	Z_R^{max}	217,607.1	159,768.4	217,223.9
	Z_R^{min}	216,418.4	138,109.8	216,044.2
	λ^*	0.7962	0.9175	0.799
	Z_M^*	684,568.2	607,552.7	684,337.6
	Z_R^*	217,364.8	157,981.5	216,986.8
	y_1^*	73.85	67.9	73.86
	y_2^*	103.18	100.2	103.21
	y_3^*	129.27	123.49	129.3

	$Z^*_{Avg}(in\ \$)$	18,038.66	15,310.68	18,026.49
	Budget utilized ($)	14060.6	13,399.5	14063.85
Our Model	Z_M^{max}	839,410	789,260	875,860
	Z_M^{min}	395,350	345,200	431,800
	Z_R^{max}	438,350	382,800	478,750
	Z_R^{min}	178,370	122,820	218,770
	λ^*	0.5	0.5	0.5
	Z_M^*	617,380	567,230	653,830
	Z_R^*	308,360	252,810	348,760
	y_1^*	0	0	0
	y_2^*	0	0	0
	y_3^*	141.6	141.6	141.6
	$Z^*_{Avg}(in\ \$)$	18,515	16,401	20,052
	Budget utilized ($)	7,080	7,080	7,080

Note: $Z^*_{Agv} = \left(\frac{Z_M^* + Z_R^*}{n}\right)$

From the obtained results in Table 2, we can see that our model gives better solutions in terms of compromised average objective value $Z^*_{Avg}(in\ \$)$. It means that total profit of the supply chain is better even though the manufacturer may have to sacrifice a little bit their benefit to share for the retailer. In addition, our model also gives better use of budget money than the Saha's solutions. Budget consumption is just about 50% of Saha's level but it still gives better objective values. In short, our model is more advantage than Saha's approach.

For generalization, we also illustrate our general case of profit-sharing supply chain coordination model with all fuzzy stochastic demand, cost, and budget. We also run an experiment with the same input data except for fuzzy stochastic values of demand, cost, and budget as presented in Table 3.

Table 3. Distribution and input parameters for fuzzy demand, cost and budget

Distribution	$E(x_i)$	$Var(x_i)$	Input Parameters
Uniform Distribution $f(\tilde{x}_i) = \frac{1}{\tilde{b}_i - \tilde{a}_i}, \tilde{b}_i \geq \tilde{x}_i \geq \tilde{a}_i$	$\left[\frac{\tilde{a}_i + \tilde{b}_i}{2}\right]$	$\left[\frac{(\tilde{b}_i - \tilde{a}_i)^2}{12}\right]$	c_{11}=(10,3,3);c_{12}=(12,4,4) c_{21} =(15,2,2);c_{22}=(16,5,5) c_{31} =(16,1,1);c_{32} =(17,2,2) demand a_1=(10,1,1); b_1=(120,10,10), a_2=(20,2,2); b_2= (160,20,20), a_3 =(30,3,3); b_3= (200,30,30) Budget: B_1 =(15000, 700,700); B_2 =(16000, 500, 500)
Exponential Distribution $f(x_i) = \tilde{\beta}_i e^{-\tilde{\beta}_i x_i}, x_i \geq 0$	$\frac{1}{\tilde{\beta}_i}$	$\left(\frac{1}{\tilde{\beta}_i^2}\right)$	Cost: β_{C1} = (0.1,0.03,0.03) β_{C2} = (0.15,0.02,0.02) β_{C3} = (0.16,0.01,0.01) Demand: β_1 = (0.015,0.001,0.001), β_2 = (0.011, 0.0008,0.0008), β_3 = (0.01, 0.0005, 0.0005) Budget: β_B = (0.0004, 0.00005, 0.00005)
Normal Distribution $f(x_i) = \frac{1}{\tilde{\sigma}_i\sqrt{2\pi}} e^{-\frac{(x_i - \tilde{\mu}_i)^2}{2\tilde{\sigma}_i^2}}$	$\tilde{\mu}_i$	$\tilde{\sigma}_i^2$	Cost: μ_{c1}= (10,3,3),σ_{c1}=(3,1,1), μ_{c2}= (15,2,2), σ_{c2}=(4,1,1) μ_{c3}= (16,1,1),σ_{c3}=(5,2,2) Demand: μ_1= (60,5,5),σ_1=(24,2,2), μ_2= (80,10,10), σ_2=(32,3,3) μ_3= (100,20,20),σ_3=(40,5,5) Budget: μ_B= (15000,700,700),σ_B=(500,50,50),

We also compare the results for different multi-objective linear programming approaches as the same way of Saha et al (2015). For ease of reading, the Global Criteria Method (GCM) and Weighted Sum Method (WSM) are also summarized here.
The GCM model is:

$$Minimize\ G(x) = \left\{\sum_{k=1}^{K}\left(\frac{z_k^{max} - z_k(x)}{z_k^{max}}\right)^p\right\}^{1/p} \quad (65)$$

Subject to $x \in X,$ (66)

Where 1≤ p <. A usual value of p is 2
The WSM model is:

$$Maximize\ U(x) = \sum_{k=1}^{K} w_k Z_k(x) \quad (67)$$

Subject to $x \in X,$ (68)

Where $\sum_{k=1}^{K} w_k = 1$ and 0≤ w_k≤ 1; k =1,2…, K

Constraint (66) or (68) is a set of given constraints of our model.

Table 4 shows the summary results of our general fuzzy stochastic model (51) – (59) using three different methods: FCPM, GCM, and WSM.

Table 4. Solution results of general fuzzy stochastic model

Solution Method	Max/Min Values and Optimal Variables	Type of distribution for stochastic demand		
		Uniform distribution	Exponential distribution	Normal distribution
FCPM	Z_M^{max}	723,245.3	734,817.467	717,979.8
	Z_M^{min}	406,142.64	296,650.8	388,859.8
	Z_R^{max}	452,005.85	332,359.2	432,080.2
	Z_R^{min}	169,555.61	56,959.2	157,414.6
	λ^*	0.459	0.498	0.4667959
	Z_M^*	551,742.2	514,788.4	542,491.7
	Z_R^*	299,244.3	194,064.8	285,627.4
	y_1^*	0	0	0
	y_2^*	174.6845	0	0
	y_3^*	0	150.65	159.5347
	$Z_{Avg}^*(in\ \$)$	17,019.73	14,177.06	16,562.382
	Budget utilized ($)	7,860.8025	7,532.5	7,976.735
GCM	G^*	0.3766807	0.4854191	0.3876186
	Z_M^*	489,190.2	444,446.4	482,648.6
	Z_R^*	364,873.2	238,660.0	342,674.1
	y_1^*	0	0	0
	y_2^*	99.63709	0	0
	y_3^*	0	102.0688	97.39232
	$Z_{Avg}^*(in\ \$)$	17,081.268	13,662.13	16,506.454
	Budget utilized ($)	4,483.6691	5,103.44	4,869.616
WSM	U^*	429,074.2	394,005	417,202
	Z_M^*	406142.6	731,050.8	676,989.4
	Z_R^*	452,005.9	56,959.20	157,414.6
	y_1^*	0	0	0
	y_2^*	0	0	0
	y_3^*	0	300	299.2000
	$Z_{Avg}^*(in\ \$)$	17,162.97	15,760.2	16,688.08
	Budget utilized ($)	0	15,000	14,960

Note: $Z_{Agv}^* = \left(\frac{Z_M^* + Z_R^*}{n}\right)$. In the case of WSM, $w_M = w_R = 0.5$.

From the obtained results in Table 4, all three methods FCPM, GCM, and WSM give quite same level the whole supply chain profit (Z_{Agv}^* value) in which WSM's results are a little bit higher than the others. All three methods also favored the manufacturer while the GCM is better in terms of the utilized budgets and balancing profits between manufacturer and retailer. In conclusion, the GCM could be the preferable method for solving the complete fuzzy stochastic supply chain coordination model.

4. CONCLUSION

We have developed new fuzzy stochastic supply chain coordination models for long-term contracts with a profit-sharing scheme. We defined two new measures of stochastic composite measures and fuzzy dominance degree to de-randomize and de-fuzzify demand, manufacturing cost and budget values. These measures will help to cover all shape characteristics as well as reflect the relative relationship between fuzzy numbers. The models also include demand variation in the objective to minimize highest risk aversion factors and avoid decision makers bias. The models are converted to the corresponding deterministic multi-objective linear programming models and solved by fuzzy compromise programming method, global criteria method, and weighted sum method. We compare our first model with the similar one of Saha et al. (2015). Our model gives better results than Saha's model in both terms of supply chain profit and budget utilization. In addition, we also developed the general case of complete fuzzy stochastic supply chain coordination model for long-term contracts with profit sharing scheme. The current models have just considered the manufacturer-retailer supply chain. In further research, we would develop fuzzy stochastic coordination models for multi-echelon supply chain system.

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