

## Study of Parametric Effects on Floating and stable Equilibrium Lengths of Composite Cylinder of Uniform Cross-Section in water

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**Abstract :** Floating and stable equilibrium conditions are pronounced in most of the fluid statics applications. In this article a uniform cross sectional wooden cylinder attached with metallic plate at the bottom placed in water is considered and variation of the floating and stable equilibrium lengths of wooden section with various parameters such as diameter of the cylinder, thickness of the metallic plate, specific gravity of the metallic plate with appropriate interval are theoretically investigated using Engineering Equation Solver (EES). Result shows that based on the application and other constraints the designer can have an option of choosing the best combination of parameters to minimize the cost.

**Keywords:** Composite cylinder, Metacenter, floating equilibrium, Stable equilibrium.

### Nomenclature:

A: Area  $m^2$

B: Buoyancy center

G: Center of Gravity

h: height  $m$

I: Moment of Inertia  $m^4$

M: Meta Center

V: Volume  $m^3$

S: Specific Gravity

l: length  $m$

Subscripts:

liq: liquid

max: maximum

min: minimum

wd: wood

m: metal

### Introduction:

In fluid statics the stability of the objects is of prime importance. The object may be either immersed or floating at the surface of the liquid. To test the stability type of equilibrium is to be considered. Hence buoyancy force as well as metacentric height will play a major role.

In this article the parametric effects on the floating and stable equilibrium lengths of a wooden composite cylinder attached with metallic plate has been studied. Various parameters considered are Specific gravity of metal, thickness of metal and diameter of the cylinder.

### System Description:

A composite solid cylinder of uniform cross section with diameter  $d$  consists of a metallic plate with specific gravity  $S_m$  of thickness  $l_m$  attached at the lower end of a wooden

portion of specific gravity  $S_{wd}$  as shown in figure 1. In the following section a mathematical model has been developed to find the theoretical limits of length of wooden portion so that composite cylinder can float in stable equilibrium in the liquid with its vertical axis [1].

### Mathematical Modeling:

To develop the mathematical model of the composite cylinder under consideration, the following assumptions are made

1. Metallic plate is firmly attached to the wooden cylinder.
2. Liquid is completely in static condition.
3. Disturbance in liquid due to placement of cylinder are negligible.

As the range for the length of wooden portion in composite cylinder is to be determined two cases emerge

- a. Floating Equilibrium and
- b. Stable Equilibrium

The first case determines the minimum length of the wooden portion and Second case determines the maximum length of the wooden portion in the composite cylinder.

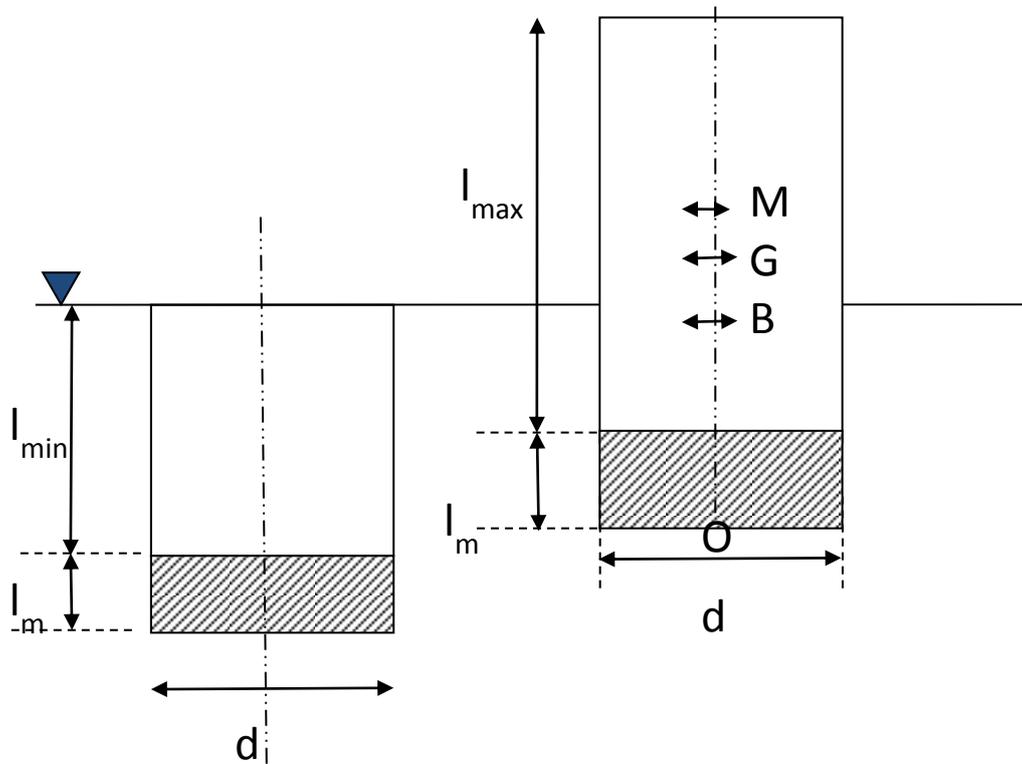


Figure 1 Composite Cylinder with floating equilibrium and stable equilibrium

**Case1: Floating Equilibrium**

The condition to be satisfied for floating equilibrium is

Weight of cylinder ≤ Weight of liquid of same volume of cylinder  
i.e

$$W_{Cylinder} = W_{wood} + W_{Metal} \tag{1}$$

$$W_{Cylinder} = \rho_w A_c (l_m S_m + l_{min} S_{wd}) \tag{2}$$

$$\tag{3}$$

Equating 2& 3 minimum length of wood section for floating equilibrium is obtained.

**Case 2: Stable Equilibrium**

For stable equilibrium metacentric height should be greater than Zero, the location of center of gravity of the composite cylinder can be found out from the first principles as follows

$$OG = \frac{A \left( \left( \frac{S_m l_m^2}{2} \right) + S_{wd} l_{max} \left( \frac{l_{max}}{2} + l_m \right) \right)}{A (S_m l_m + S_{wd} l_{max})}$$

On simplification

$$OG = \frac{\left( \left( \frac{0.5 S_{wd} l_{max}^2}{S_m l_m} \right) + \left( \frac{S_{wd} l_{max}}{S_m} \right) + 0.5 l_m \right)}{1 + \left( \frac{S_{wd} l_{max}}{S_m l_m} \right)} \tag{4}$$

The submerged height  $h$  can be found out by equating Weight of the cylinder and Buoyancy force

$$A(S_m l_m + S_{wd} l_{max}) = S_{liq} Ah$$

$$h = \frac{S_m l_m \left( 1 + \frac{S_{wd} l_{max}}{S_m l_m} \right)}{S_{liq}}$$

The location of center of Buoyancy can be expressed as  $OB=h/2$  and Substituting  $h$  from equation (4) in

$$BG = OG - OB \tag{6}$$

The location of Meta Centre  $M$  above center of Buoyancy  $B$  can be found out by

$$MG = BM - BG \tag{7}$$

$$BM = \frac{I}{V}$$

$l_{max}$  can be found out by equating  $MG$  to zero the resulting value limits the length of the Wooden section for stable equilibrium.

**Parameters considered:** The parameters considered here are specific gravity of metal, specific gravity of wood, Metal Plate thickness and diameter of the cylinder.

**Table 1 Range of the Parameters considered**

Parameter	Range	Step
Specific Gravity of metal	2-8	+1
Metal Plate Thickness	5-50 mm	+5 mm
Diameter of cylinder	50-200 mm	Initially +10mm Then +25 mm

There will be possible four cases has been considered as mentioned below

**Table 2 Possible cases of study**

Case	Plate Thickness	Diameter of cylinder	Solved for
A.	Fixed	Varying	Minimum Length
B.	Fixed	Varying	Maximum Length
C.	Varying	Fixed	Minimum Length
D.	Varying	Fixed	Maximum Length

**Results and Discussions:**

Using the above mathematical modeling and the range of the parameters under study theoretically investigated using Engineering Equation Solver (EES) [EES]. The results obtained are discussed in this section under three categories

**A. Minimum Length of wooden section for fixed metallic thickness :**

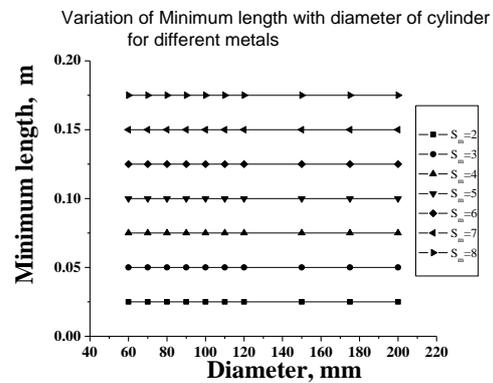


Figure 2 Variation of Minimum length with diameter of cylinder for constant metal thickness

Fig 2 Shows the Variation of minimum length of the wooden section with diameter of cylinder for different metals for the same thickness of the metallic plate. It can be seen that, the minimum length of the wooden section is independent of diameter of the composite cylinder. However minimum length of the wooden section is larger for the heavy metals in comparison with lighter metals. This is due to fact that for the heavy metals

**B Maximum Length of wooden section for fixed metallic thickness :**

Fig 3 Shows the Variation of maximum length of the wooden section with diameter of cylinder for different metals for the same thickness of the metallic plate. It can be seen that, the maximum length goes on increasing as the diameter of the composite cylinder increases for the same metal. However, it can be observed that the maximum length for the lighter metal at larger diameter equals the maximum length of the wooden section for the heavier metal with smaller diameter. This has been shown with dotted lines in Figure 3.

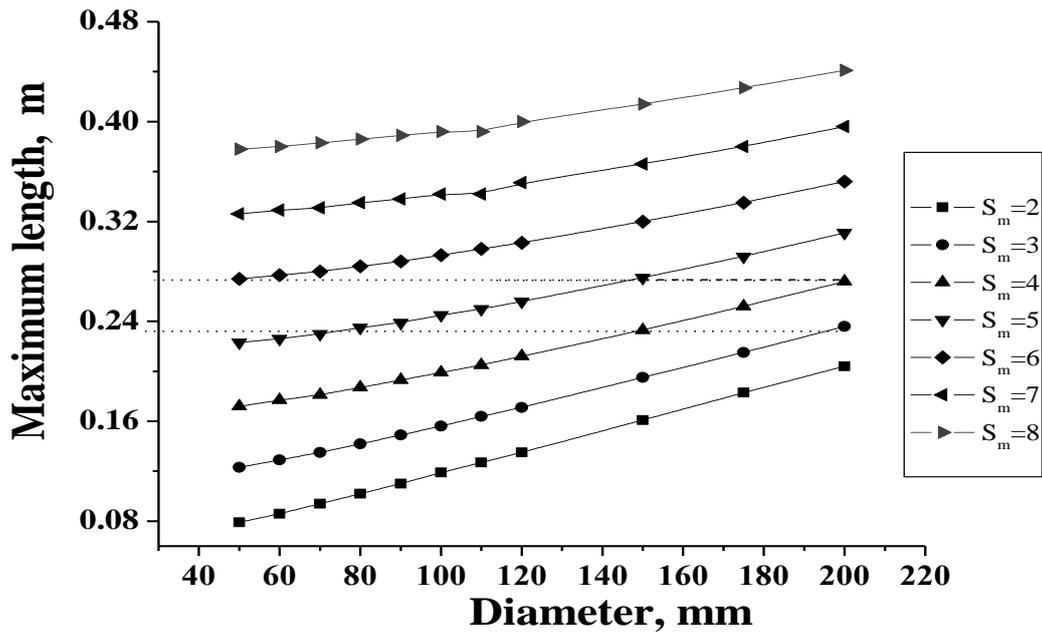


Figure 3 Variation of Maximum length with diameter of cylinder for constant metal thickness.

**A. Minimum Length of wooden section for varying metallic thickness :**

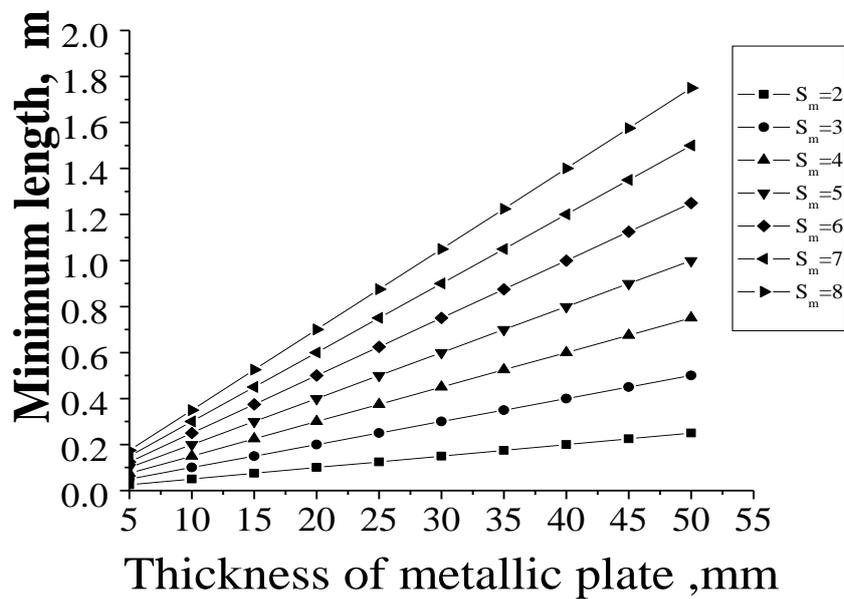


Figure 4 Variation of Minimum length with metal plate thickness for constant cylinder diameter

Fig 4 shows the variation of minimum length of the wooden section with thickness of the metal at a given diameter of cylinder for different metal specific gravity. It can be observed that for a given diameter of the cylinder, the minimum length of the cylinder goes on increasing

linearly with thickness of the metallic plate. Also as the heavier metallic plate is used the minimum length of wooden section goes on increasing as can be seen from the Figure 4.

**B. Maximum Length of wooden section for varying metallic thickness :**

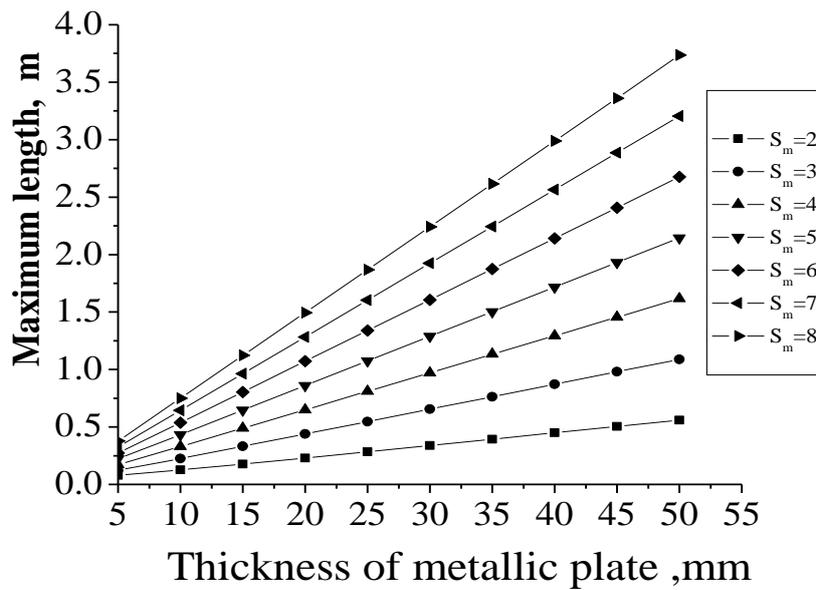


Figure 5 Variation of Maximum length with metal plate thickness for constant cylinder diameter

Fig 5 shows the variation of maximum length of the wooden section with thickness of the metal at a given diameter of cylinder for different metal specific gravity. It is also clear from the figure that the maximum length linearly increases. It is also noticeable that for the particular diameter of the cylinder the maximum limit of the wooden section goes on increasing with heavier relative density of the metal.

### CONCLUSIONS

With the presented results of Parametric Effects on Floating and stable Equilibrium Lengths of Composite Cylinder of Uniform Cross-Section in water following conclusions can be drawn

- i. For a given thickness of metallic plate, the minimum length of the wooden section for the equilibrium is independent of the diameter of the cylinder, but strongly dependent on the density of metallic plate.
- ii. For a fixed thickness of metallic plate, the maximum length goes on increasing as the diameter of the composite cylinder increases for the same metal.
- iii. Never the less maximum length for the lighter metal at larger diameter equals the

maximum length of the wooden section for the heavier metal with smaller diameter.

- iv. For the given diameter of the cylinder the minimum and maximum lengths of the wooden section vary in similar manner and provide the option for the designer to choose between long wooden section with lighter metal or shorter one with heavier metal depending on the application and other constraints because the maximum length for a lighter metal at larger diameter equals that for a heavier metal with smaller diameter.
- v. Although the result presented here are indicative and shown for a particular value of diameter of cylinder and thickness of the metallic plate similar results yield with different values. Hence parametric effects can be generalized in nature of variations.

### REFERENCES:

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