Available online at www.ijapie.org

International Journal of Advanced Production and Industrial Engineering

IJAPIE-SI-OSCM 702 (2017) 04–06

Special Issue on

OPERATIONS & SUPPLY CHAIN MANAGEMENT

National Income in the form of a Complex Dynamic system

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Abstract : National income is a function of several variables mainly consumption expenditure, govt expenditure, investment expenditure and net export (export-import). In this paper a system of recurrence relations are defined with the use of some suitable constant coefficients and these relations are fitted to a linear National income difference equation. The solution of this linear difference equation is obtained by the application of Z-transform and inverse Z-transform under suitable domain specifications. The obtained solution can be numerically computed by numerical method algorithm.

Keywords: National income, Consumption value, govt expenditure, export-import cost price, Investment cost price, z-transform, Inverse z-transform, recurrence relation, difference equation

I Introduction

National income has the key role in the decision analysis on consumption-production prices and quantities, government expenditures, export-import cost prices and quantities, and investment price. Also the national income function depends on the variables such as consumption-production prices, government expenditures, export-import cost prices, and investment price in an implicit form. Economist studied the relationship of these functions in the definition of gross domestic product (GDP) in three specifications such as GDP with the relation of production, GDP with the relation of expenditure and GDP with the relation of income. Their observation is that the three specifications of GDP should be equivalent. The direct production and consumption relation of GDP is an implicit function of national income and investment. The expenditure relation with GDP is also a implicit function of national income. Finally, the national income relation with GDP is a financial mathematics concerned topic. The control of variables is a topic of Financial engineering.

In this paper we defined a system of recurrence relations to represent the implicit linear behaviour of the variables used in the measure of national income [3,4,5] and unite these relations in the form of a difference equation. The solution of the defined difference equation is determined by the application of Z-transform and inverse Z-transform for the variable z within a determined domain. The solution to the difference equation that we have defined could be calculated by the application of generating function rather application of Z-transform and inverse Z-transform. But the Z-transform is more suitable in the control design of the implicit variables.

The presentation of this paper is as follows, article I is introduction and motivation of research, article II contain the outlines of the research such as description of existing definition, article III consists of formulation and definition of the present research article, article IV contains Restrictions and specification of Initial constants for the study of optimality and article V contains conclusion of the topic and applications

II Description of existing definition

The Z-transform has an important role in the field of Communication Engineering and Control engineering at the stages of analysis and representation of discrete time linear shift invariance system. Particular mathematical analysis is that when the continuous signals are sampled, the discrete-time functions arise. The sequence of discrete functions is either defined for all nonnegative integers or for all integers. If the sequence of discrete functions is defined for all nonnegative integers the concerned Z-transform is known as One-sided Z-transform, the Z-transform of the sequence of discrete functions defined for all integers is named as two-sided Z-transforms

In our problem solution we will use one sided Z-transform and its inverse Z-transform . For this reason we cite the definition of one sided Z-transform.[1][2]

Definition Z-transform: Let $\{f(k)\}$ be a sequence defined for all nonnegative integer k. Then the Z-transform of $f(k)$ is $Z\big(f(k)\big) = F(z) = \sum_{n=0}^{\infty} f(k)z^{-n}$, where z is an arbitrary complex number and Z is an operator of Z-transform.

The infinite series of Right hand sides of the definition's mentioned equation will be convergent only for certain values of z depending on sequence $\{f(k)\}$

Definition inverse Z-transform: Let $F(z) = Z\big(f(k)\big) = \sum_{n=0}^{\infty} f(k)z^{-n}$ be the Z-transform of the sequence $\{f(k)\}$ then inverse Z-transform of $F(z)$ is$Z^{-1}\big(F(z)\big) = \{f(k)\}$

Z-transform has some specific properties including change of scale property, properties for multiplication, or division of constants to the sequence, shifting property, and linearity property. We cite the definition of Linearity property that is used in the problem solving.

Definition Linearity of Z-transform: Let $\{f_1(k)\}$ and $\{f_2(k)\}$ be two sequences, a and b are two constants then Z-transform of $Z(a\{f_1(k)\}+b\{f_2(k)\}) = aZ(\{f_1(k)\}) + bZ(\{f_2(k)\})$

III PROBLEM FORMULATION AND DEFINITION OF THE PRESENT RESEARCH

The following difference Equations representing the relation of National income y_n, Consumption value c_n, Govt expenditure g_n, export-import cost price $(x-t)_n$ and investment cost price i_n variables

$$y_{n+1} = c_n + g_n + (x-t)_n + i_n \quad (1)$$
$$c_n = k_1 + m_1 y_n + m_2 i_n; \quad (2)$$
$$g_n = k_2 + m_3 y_n; \quad (3)$$
$$(x-t)_n = k_3 + m_4 y_n + m_5 c_n \quad (4)$$
$$i_n = m_6(y_{n+1} - y_n), \quad (5)$$

where $k_1, k_2, k_3 \geq 0$ and $m_j \in (0,1)$ for $j = 1,2,\dots,6$. The variable $y_{(n+1)}$ represents the value of the national income after the one step ahead of the time instant n. Substituting the right hand side of the variable i_n from Equation (5) in the Equation (2) the variable c_n can be expressed as the following difference equation

$$c_n = k_1 + m_1 y_n + m_2 m_6(y_{n+1} - y_n) \quad (6)$$

Substituting the Right hand side of the variablesg_n, $(x-t)_n$ i_n from Equations (3), (4) and (5) in the equation (1) we get the following difference equation

$$y_{n+1} = (1+m_5)c_n + k_2 + m_3 y_n + k_3 + m_4 y_n + m_6(y_{n+1} - y_n)$$

Again substituting the Right hand side of the variable c_n from equation (6) in the above equation we get

$$(1 - ((1+m_5)m_2 + 1)m_6))y_{n+1} = [(1+m_5)k_1 + k_2 + k_3] + [(1+m_5)(m_1 - m_2 m_6) + m_3 + m_4 - m_6]y_n \quad (7)$$

The above difference equation is a linear equation with the constants $(1 - ((1+m_5)m_2 + 1)m_6))$, $((1+m_5)k_1 + k_2 + k_3)$ and $((1+m_5)(m_1 - m_2 m_6) + m_3 + +m_4 - m_6)$. The problem is to solve this difference equation with appropriate specification of the constants. We apply z-transform to solve this difference equation. For this we determine the z-transform of the sequences y_{n+1}, y_n and $\{1\}$ separately as follows

The z-transform of y_{n+1} is $Z(y_{n+1}) = \sum_{n=0}^{\infty} y_{n+1} z^{-n} = \sum_{n=0}^{\infty} y_{n+1} z^{-n-1} z = z\sum_{n=0}^{\infty} y_{n+1} z^{-(n+1)} = zY(z) - y_0 z$

The z-transform of y_n is $Z(y_n) = \sum_{n=0}^{\infty} y_n z^{-n} = Y(z)$ and the z-transformation of the sequence $\{1\}$ is $Z(\{1\}) = \sum_{n=0}^{\infty} z^{-n} = \frac{z}{z-1}$

Using linearity property of z-transform we obtain the following equation

$$(1 - ((1+m_5)m_2 + 1)m_6))Z(y_{n+1}) - [(1+m_5)(m_1 - m_2 m_6) + m_3 + m_4 - m_6]Z(y_n) = [(1+m_5)k_1 + k_2 + k_3]\, Z(\{1\})$$

Or, $(1 - ((1+m_5)m_2 + 1)m_6))(zY(z) - y_0 z) - [(1+m_5)(m_1 - m_2 m_6) + m_3 + m_4 - m_6]Y(z) = [(1+m_5)k_1 + k_2 + k_3]\, \frac{z}{z-1}$

Or, $((1 - ((1+m_5)m_2 + 1)m_6))z - [(1+m_5)(m_1 - m_2 m_6) + m_3 + m_4 - m_6])Y(z) = \big(1 - \big((1+m_5)m_2 + 1\big)m_6\big)y_0 z + [(1+m_5)k_1 + k_2 + k_3]\, \frac{z}{z-1}$

Or, $$Y(z) = \frac{(1-((1+m_5)m_2+1)m_6)y_0 z}{(1-((1+m_5)m_2+1)m_6)z - [(1+m_5)(m_1-m_2m_6)+m_3+m_4-m_6])} + \frac{[(1+m_5)k_1 + k_2 + k_3]z}{((1 - ((1+m_5)m_2 + 1)m_6)z - [(1+m_5)(m_1 - m_2 m_6) + m_3 + m_4 - m_6])(z-1)}$$

Or, $$Y(z) = \frac{y_0 z}{\left(z - \frac{[(1+m_5)(m_1-m_2m_6)+m_3+m_4-m_6]}{\left(1-\left((1+m_5)m_2+1\right)m_6\right)}\right)} + \frac{\left(\frac{[(1+m_5)k_1+k_2+k_3]}{(1-((1+m_5)m_2+1)m_6)}\right)z}{\left(z - \frac{[(1+m_5)(m_1-m_2m_6)+m_3+m_4-m_6]}{(1-((1+m_5)m_2+1)m_6)}\right)(z-1)}$$

Or, $$Y(z) = \frac{y_0 z}{\left(z - \frac{[(1+m_5)(m_1-m_2m_6)+m_3+m_4-m_6]}{\left(1-\left((1+m_5)m_2+1\right)m_6\right)}\right)} - \frac{\left(\frac{[(1+m_5)k_1+k_2+k_3]((1+m_5)(m_1-m_2m_6)+m_3+m_4-m_6)}{(1-(1+m_5)m_1-m_3-m_4)}\right)z}{\left(z - \frac{[(1+m_5)(m_1-m_2m_6)+m_3+m_4-m_6]}{(1-((1+m_5)m_2+1)m_6)}\right)} + \frac{\left(\frac{[(1+m_5)k_1+k_2+k_3]}{(1-(1+m_5)m_1-m_3-m_4)}\right)z}{(z-1)}$$

Or, $$Y(z) = \frac{(y_0 - s_1)z}{(z-a)} + \frac{s_2 z}{(z-1)}$$

Where $= \frac{[(1+m_5)(m_1-m_2m_6)+m_3+m_4-m_6]}{(1-((1+m_5)m_2+1)m_6)}$, $s_1 = \frac{[(1+m_5)k_1+k_2+k_3]((1+m_5)(m_1-m_2m_6)+m_3+m_4-m_6)}{(1-(1+m_5)m_1-m_3-m_4)}$ and $s_2 = \left(\frac{[(1+m_5)k_1+k_2+k_3]}{(1-(1+m_5)m_1-m_3-m_4)}\right)$

The constants $k_1, k_2, k_3, m_1, m_2, m_3, m_4, m_5, m_6$ and the initial condition y_0 are to be known for determining the numerical value of the function $Y(z)$. If $|z| > \lceil a \rceil$, by the inverse transformation we obtain the solution to the difference equation (7) as follows

$$y_n = (y_0 - s_1)a^n + s_2 1^n \text{ for } n = 0,1,2,\dots \quad (8)$$

IV Restrictions and specification of Initial constants for the study of optimality

The solution y_n represents the national income of nth instant and this will be maximum if the followings conditions mentioned in the cases 1 and 2 are to be satisfied.

Case1: $y_0 > s_1, a > 1$and $s_2 > 0$

Case 2: In case $y_0 < s_1$, a should be less than zero and optimality can occur for odd n and $s_2 > 0$

If Case1 conditions satisfy, the restriction $a > 1$ implies $m_1 + m_5 m_1 + m_3 + m_4 > 1$ and the restriction $s_2 > 0$ implies $\frac{[(1+m_5)k_1+k_2+k_3]}{(1-(1+m_5)m_1-m_3-m_4)} > 0$ or $(1 + m_5)m_1 + m_3 + m_4 < 1$ as $m_5, k_1, k_2, k_3 > 0$. The above two derived inequalities imply $(1 + m_5)m_1 + m_3 + m_4 = 1$. There is no loss of generality if we assume the restriction $\lim_{R\to\infty} \frac{1}{R} = m_5$ and $m_1 + m_3 + m_4 = 1$, this is possible as we have defined $m_1, m_3, m_4, m_5 \epsilon (0,1)$ and $k_1, k_2, k_3 \geq 0$

V CONCLUSIONS

The Z-transform function $Y(z) = \frac{(y_0 - s_1)z}{(z-a)} + \frac{s_2 z}{(z-1)}$ is the sum of two linear fractional transformations with poles at $z = a$ and $z = 1$. Each linear fractional transformation converts z-domain to the complex w domain. The stable poles are in the left half plane, unstable poles are in the right half plane and marginal stable poles are in vertical imaginary axis.

The discussed Z-transform can be fitted to controller design which can control the variables used in actual function of national income to the desired level.

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